

Paper Reference 9FM0/4A
Pearson Edexcel Level 3 GCE
Further Mathematics

Advanced
PAPER 4A: Further Pure Mathematics 2
Monday 23 June 2025 – Afternoon
Time: 1 hour 30 minutes

Question Paper

**THIS DOCUMENT MUST BE
RETURNED TO PEARSON AT THE END
OF THE EXAMINATION.**

**In the boxes below, write your name,
centre number and candidate number.**

Surname					
Other names					
Centre Number					
Candidate Number					

YOU MUST HAVE

**Mathematical Formulae and Statistical
Tables (Green), calculator**

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

**Candidates may use any
calculator allowed by Pearson
regulations. Calculators
must not have the facility for
symbolic algebra manipulation,
differentiation and integration,
or have retrievable mathematical
formulae stored in them.**

Turn over

INSTRUCTIONS

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or in the separate Diagram Booklet – there may be more space than you need.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

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INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 9 questions in this Question Paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams.

Turn over

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to the table for Question 1 in the Diagram Booklet.

It shows an incomplete Cayley table.

The set

$S = \{1, 3, 5, 9, 11, 13\}$ forms

the group G , under the operation multiplication modulo 14

- (a) Complete the Cayley table in the Diagram Booklet for the group G
There are twelve spaces to fill.

(3 marks)

- (b) Write down a subgroup of G of order 2

(1 mark)

(continued on the next page)

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1. continued.

**(c) Refer to the table for
Question 1(c) in the Diagram
Booklet.**

**It shows another Cayley table.
The group H is defined by the
Cayley table in the Diagram
Booklet.**

**Show that G and H are
isomorphic.
(3 marks)**

(Total for Question 1 is 7 marks)

2.

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$$

(a) Determine the eigenvalues of matrix **A**
(3 marks)

(b) Hence determine an orthogonal matrix **P** and a diagonal matrix **D** such that

$$\mathbf{D} = \mathbf{P}^T \mathbf{A} \mathbf{P}$$

(5 marks)

(Total for Question 2 is 8 marks)

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3. The circle **C** has equation

$$|z - 5 + 4i| = 2|z - 2 + i|$$

where

$$z = x + iy$$

(a) Show that an equation for **C** is

$$x^2 - 2x + y^2 - 7 = 0$$

(3 marks)

(continued on the next page)

3. continued.

The half–line with equation

$$\arg(z - a) = -\frac{3\pi}{4}$$

where a is a real constant, is a tangent to C

**(b) Determine the value of a
(6 marks)**

(Total for Question 3 is 9 marks)

4. A sequence is defined by

$$u_{n+2} = 6u_{n+1} - 8u_n \quad n \geq 1$$

$$u_1 = 399 \cdot 6 \quad u_2 = 798 \cdot 4$$

(a) Prove by induction that, for $n \in \mathbb{N}$

$$u_n = 200(2^n) - \frac{1}{10}(4^n)$$

(6 marks)

(continued on the next page)

4. continued.

(b) Hence determine the value of the first negative term of the sequence.

(Solutions relying entirely on calculator technology are not acceptable.)

(3 marks)

(Total for Question 4 is 9 marks)

5. (i) A password is to be created from 5 characters using the following criteria

- one capital letter**
- 3 distinct single digits**
- one of the four symbols
@ £ \$ &**

Determine how many different passwords can be created if

**(a) the characters are in the
order of the bullet points
above,
(2 marks)**

(continued on the next page)

Turn over

5. (i) continued.

(b) the characters can be in any order.

(1 mark)

(ii) Use Fermat's Little Theorem to determine the least positive residue of 4^{50} modulo 13

(3 marks)

(iii) Determine the total number of different, positive ODD numbers, less than 1000, that contain the digit 2

(4 marks)

(Total for Question 5 is 10 marks)

Turn over

6. $I_n = \int_0^2 (4 - x^2)^n dx$

(a) Prove that, for $n \geq 1$

$$I_n = \frac{8n}{2n+1} I_{n-1}$$

(4 marks)

(continued on the next page)

6. continued.

(b) Using the result from part (a) and showing all stages of your working, determine the value of n for which

$$\mathbf{I_n = \frac{65536}{315}}$$

(Solutions relying entirely on calculator technology are not acceptable.)

(4 marks)

(Total for Question 6 is 8 marks)

7. (i) Explain why the congruence equation

$$21x \equiv 11 \pmod{36}$$

has no solutions.

(2 marks)

- (ii) Use modular arithmetic to solve the congruence equation

$$36y \equiv 192 \pmod{40}$$

(4 marks)

(Total for Question 7 is 6 marks)

8. Refer to Diagram 1 and Diagram 2 for Question 8 in the Diagram Booklet.

Diagram 1 shows a satellite dish.

Diagram 2 shows a sketch of the curve **C** with equation

$$y^2 = Ax \qquad 0 \leq x \leq 10$$

where **A** is a positive constant.

(continued on the next page)

8. continued.

The curved inner surface of the satellite dish is modelled by the surface of revolution formed by rotating curve C through π radians about the X -axis.

The inner surface of the satellite dish has

- a largest diameter of 60 cm**
- a depth of 10 cm**

as shown in Diagram 1

(continued on the next page)

Turn over

8. continued.

**(a) Determine the value of A
(2 marks)**

**(b) Using algebraic integration,
determine, in cm^2 , the area of
the curved inner surface of the
satellite dish, according to the
model.**

**Give your answer to 2 significant
figures.
(7 marks)**

**(c) State a limitation of the model.
(1 mark)**

(Total for Question 8 is 10 marks)

Turn over

9. The transformation T from the z -plane to the w -plane, where $z = x + iy$ and $w = u + iv$ is given by

$$w = \frac{z}{z + 2i} \quad z \neq -2i$$

The circle with equation $|z| = 4$ is mapped by T onto the circle C

(continued on the next page)

9. continued.

Determine

**(i) the coordinates of the centre
of C**

**(ii) the radius of C
(8 marks)**

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER